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Testing the FPS approach in $d = 1$

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ABSTRACT: We apply the approach of S. Ferrara, M. Porrati and A. Sagnotti [1] to the one dimensional system described by the $N = 2, d = 1$ supersymmetric action for two particles in which one of $N = 1$ supersymmetries is spontaneously broken. Using the nonlinear realization approach we reconsider the system in the basis where only one superfield has the Goldstone nature while the second superfield can be treated as the matter one, being invariant under transformations of the spontaneously broken $N = 1$ supersymmetry. We establish the transformations relating the two selected FPS-like cases with our more general one, and find the field redefinitions which relate these two cases. Thus we demonstrate, at least in one dimension, that the only difference between two FPS cases lies in the different choice of the actions, while the supermultiplets specified by the FPS-like constraints are really the same. Going further with the nonlinear realization approach, we construct the most general action for the system of two $N = 1$ superfields possessing one additional hidden spontaneously broken $N = 1$ supersymmetry. The constructed action contains two arbitrary functions and reduces to the FPS actions upon specification of these functions. Unfortunately, the exact form of these functions corresponding to FPS actions is not very informative and gives no explanation on why the FPS cases are selected.

KEYWORDS: Supersymmetry Breaking, Extended Supersymmetry, Superspaces

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1 Introduction

The key idea of Bagger and Galperin approach in constructing the $N = 2, D = 4$ supersymmetric Born-Infeld theory was to embed $N = 1$ vector supermultiplet W_α along with $N = 1$ chiral one X into $N = 2$ chiral supermultiplet (X, W_α) in such a way that the transformations of additional hidden $N = 1$ supersymmetry with parameters $\eta_\alpha, \bar{\eta}_{\dot{\alpha}}$

$$\delta(W)_\alpha = \left(1 - \frac{1}{4}\overline{D}^2\overline{X}\right)\eta_\alpha - i\partial_{\alpha\dot{\alpha}}X\bar{\eta}^{\dot{\alpha}}, \quad \delta X = -2(W)^\alpha\eta_\alpha, \quad (1.1)$$

describe its spontaneous breaking due to the presence of the constant term in the transformation law of W_α . In view of the property of $N = 1$ vector supermultiplet

$$D^2W_\alpha \sim \partial_{\alpha\dot{\alpha}}\overline{W}^{\dot{\alpha}}$$

the trivial action

$$S = \int d^4x d^2\theta X \quad (1.2)$$

becomes invariant under transformations (1.1). The action (1.2) acquires a real meaning after introducing the additional constraint, invariant under (1.1) transformations

$$W \cdot W + X \left(m - \frac{1}{4}\overline{D}^2\overline{X}\right) = 0, \quad (1.3)$$

which can be solved in order to express X in terms of the superfields $W_\alpha, \bar{W}_{\dot{\alpha}}$ [2]. The bosonic core of the resulting action is just the Born-Infeld action [2]. Thus, the $N = 2$ supersymmetric Born-Infeld theory describes the partial spontaneous breaking of $N = 2$ supersymmetry to $N = 1$ one with a $N = 1$ vector supermultiplet as the corresponding Goldstone supermultiplet.

In the recent paper [1] S. Ferrara, M. Porrati and A. Sagnotti proposed the generalization of J. Bagger and A. Galperin construction to the cases of several $N = 1$ vector supermultiplets. The FPS approach includes two basic ingredients. First, the nonlinear constraint (1.3) is generalized to be

$$d_{abc} \left(W_b \cdot W_c + X_b \left(m_c - \frac{1}{4} \bar{D}^2 \bar{X}_c \right) \right) = 0, \quad (1.4)$$

where $W_{a\alpha}, X_a$ are n -copies of $N = 1$ vector and chiral multiplets, d_{abc} are totally symmetric constants and m_a is a set of constants. The realization of hidden $N = 1$ supersymmetry is the same as in the Bagger and Galperin case

$$\delta W_{a\alpha} = \left(m_a - \frac{1}{4} \bar{D}^2 \bar{X}_a \right) \eta_\alpha - i \partial_{a\dot{\alpha}} X_a \bar{\eta}^{\dot{\alpha}}, \quad \delta X_a = -2 W_a^\alpha \eta_\alpha. \quad (1.5)$$

The invariance of the constraint (1.4) with respect to the hidden $N = 1$ supersymmetry transformations (1.5) introduced the additional constraint (which is trivially satisfied in the case of one supermultiplet)

$$d_{abc} W_{b\alpha} X_c = 0. \quad (1.6)$$

The constraints (1.4), (1.6) can be also solved expressing the bosonic $N = 1$ chiral superfields X_a in terms of the $N = 1$ vector supermultiplets $W_{a\alpha}$.

The next nontrivial step of FPS approach is the structure of the corresponding action

$$S = \int d^4 x d^2 \theta \left[e^a X_a + C_{ab} \left(W_a \cdot W_b + X_a \left(m_b - \frac{1}{4} \bar{D}^2 \bar{X}_b \right) \right) + c.c. \right], \quad (1.7)$$

$$e^a = \text{const}, \quad C_{ab} = C_{ba} = \text{const}.$$

The additional term with C_{ab} is also invariant with respect to transformations (1.5). Such a term does not exist in the case of one supermultiplet, but it proved to be essential in the cases of several supermultiplets [1].¹ The action (1.7) with X_a being the solution of the constraints (1.4), (1.6) is treated in [1] as the many-field extension of $N = 2$ Born-Infeld theory.

In paper [1] the detailed analysis of $n = 2$ case was presented, which can be divided into two subcases:

- $d_{111} = 1, d_{112} = -1 \quad I_4 = 0,$
- $d_{111} = 1, d_{122} = -1 \quad I_4 > 0,$

¹We are thankful to S. Ferrara, M. Porrati and A. Sagnotti for the correspondence concerning the essentiality of this term.

where I_4 is a quartic invariant, discussed in [1] (see also the refs. therein). Then the analysis of $n = 3$ case has been performed in [3].

The above, quite short sketch of the FPS approach is enough to raise two interesting questions

- whether the supermultiplets with different constants d_{abc} are really different?
- whether the action (1.7) is unique as it happened in $n = 1$ case?

The reasons for raising the first question are the following. After resolving the constraints FPS (1.4), (1.6) we will have in the theory n copies of $N = 1$ vector multiplets $W_{a\alpha}$ with highly nonlinear transformation properties under hidden $N = 1$ supersymmetry (1.5). The presence of the constants m_a in (1.5) at first sight means that all of them have Goldstone nature. But it cannot be so, because the spontaneous breaking of $N = 2$ to $N = 1$ supersymmetries accompanied by the presence in the theory of only one Goldstone superfield [4–7]. Moreover, as it follows from an alternative construction of $N = 2$ supersymmetric Born-Infeld action within the nonlinear realization approach discussed in [2], it should exist a special basis in which only one superfield is Goldstone superfield, while the remaining superfields have to be matter superfields with the trivial transformation properties with respect to broken supersymmetry. In view of the relation between linear and nonlinear realizations of partially broken supersymmetries [8, 9], one should expect that any linear realization of broken supersymmetry can be transformed by a proper highly nonlinear, but invertible field redefinition, to the *same* nonlinear realization in this basis. Of course, the nonlinear realization approach is less useful for constructing of superfield actions, but the claim that different choices of the constants d_{abc} in the basic constraints (1.4), (1.6) must be equivalent, should be carefully analyzed.

The second question about uniqueness of the action we are raising is also related with existence of the special basis in which we have one Goldstone and an arbitrary number of matter superfields. In such a basis one may easily construct the general action which will have a functional freedom. Thus, it has to be quite interesting to analyze this general action and understand the reasons that just select the FPS actions.

Unfortunately, the analysis of $N = 2$ extended Born-Infeld theory is quite involved, and the ideological part is hidden behind complicated calculations. Fortunately enough the FPS approach, as we already stressed in [10], is not limited by application to Born-Infeld theory only. Instead, it will work perfectly for many other systems with partial breaking of supersymmetry. The simplest of all such examples is provided by a theory describing partial breaking of $N = 2$ to $N = 1$ supersymmetry in one dimension - the textbook case considered in [11]. The FPS generalization of such a system, being quite simple, will inherit all essential properties of the FPS construction. In the present paper we are exploring the generalization of FPS construction for this case. In the next section we remind the description of $N = 2 \rightarrow N = 1$ partial breaking of supersymmetry in $d = 1$ [11] and then extend it to the many-particle case following to FPS approach. In section 3 we explicitly establish the transformations that relate two different $n = 2$ cases with the special basis discussed above, as well as the explicit relations between them. In section 4

we construct the general action for such a system containing two arbitrary functions and provide the explicit form of these functions reducing the general action to FPS-like ones. Finally, in Conclusion we summarize our results.

2 Partial breaking in one dimension: superparticle action and its generalization

Let us start, following [11], with a simple example of superparticle model describing $N = 2 \rightarrow N = 1$ partial breaking of global supersymmetry in $d = 1$. In this case, the anticommutation relation of $N = 1, d = 1$ Poincaré superalgebra reads

$$\{Q, Q\} = 2P. \quad (2.1)$$

The coordinates (t, θ) of $N = 1$ superspace satisfy the following rules of complex conjugation: $t^\dagger = -t, \theta^\dagger = \theta$. Next, we define the bosonic and fermionic superfields $v(t, \theta)$ and $\psi(t, \theta)$ related as

$$\psi = \frac{1}{2} Dv, \quad (v^\dagger = -v, \psi^\dagger = \psi), \quad (2.2)$$

where the spinor derivative D obeys the relation²

$$D = \frac{\partial}{\partial \theta} + \theta \partial_t, \quad \{D, D\} = 2\partial_t. \quad (2.3)$$

2.1 Linear realization: one particle case

To realize an additional spontaneously broken $N = 1$ supersymmetry we introduce, in full analogy with [2], two spinor superfields ψ and ν transforming under the S -supersymmetry as

$$\delta\psi = \varepsilon(1 - D\nu), \quad \delta\nu = \varepsilon D\psi. \quad (2.4)$$

The presence of the constant shift in (2.4) suggests the interpretation of ψ as the Goldstone fermion accompanying the $N = 2 \rightarrow N = 1$ breaking. The superfield ν , due to its transformation properties under the S -supersymmetry (2.4), may be chosen as a Lagrangian density, since the integral

$$S = \int dt d\theta \nu \quad (2.5)$$

is invariant with respect to both broken and unbroken supersymmetries. To be meaningful, the action (2.5) should be accompanied an additional constraint, which is invariant under transformations (2.4) and which, by analogy with (1.3), will allow to express the superfield ν through the superfield ψ . One may easily check, that the corresponding invariant constraint looks quite similar to (1.3) to be [11]

$$\psi D\psi - \nu(1 - D\nu) = 0 \quad (2.6)$$

with evident solution given by

$$\nu = \frac{2\psi D\psi}{1 + \sqrt{1 - 4(D\psi)^2}}. \quad (2.7)$$

²The time derivative of any variable f denoted as: $\dot{f} = \partial_t f$.

Note, that from (2.6) follows

$$\psi\nu = 0. \quad (2.8)$$

The bosonic core of the action (2.5) with the integrand (2.7) corresponds to the action of particle in $d = 1$

$$S_{\text{bos}} = \frac{1}{2} \int dt \left(1 - \sqrt{1 - (\dot{v})^2} \right). \quad (2.9)$$

Thus, this simplest system demonstrate a close analogy with the Bagger and Galperin construction of the supersymmetric Born-Infeld action.

2.2 Generalization to the multiparticle case

The above considered case can be easily extended to that of an arbitrary number of $N = 1$ superfields strictly following the FPS approach. Doing so, we firstly introduce a set of n superfields ψ_a and ν_a transforming under implicit $N = 1$ supersymmetry as

$$\delta\psi_a = \varepsilon(m_a - D\nu_a), \quad \delta\nu_a = \varepsilon D\psi_a, \quad (2.10)$$

where m_a are arbitrary constants. Now, again in full analogy with FPS approach, we impose the following generalization of the constraints (2.6)

$$d_{abc}(\psi_b D\psi_c - \nu_b(m_c - D\nu_c)) = 0. \quad (2.11)$$

The invariance of these constraints under broken supersymmetry (2.10) leads to an additional restriction

$$\partial_t(d_{abc}\psi_b\nu_c) = 0, \quad (2.12)$$

which can be reformulated as

$$d_{abc}\psi_b\nu_c = 0, \quad (2.13)$$

assuming that the constant of integration is equal to zero. Finally, the FPS-like generalization of the action (2.5) reads

$$S = \int dt d\theta [e^a \nu_a + C_{ab}(\psi_a D\psi_b - \nu_a(m_b - D\nu_b))], \quad (2.14)$$

where e^a and $C_{ab} = C_{ba}$ are arbitrary real constants.

3 Standard basis for $n = 2$ cases

Up to now we have just repeated the basic steps of the FPS approach applying it to the one dimensional case. Let us now consider in detail a system of two superparticles.

In the standard approach of the spontaneous breaking of supersymmetry the two particle model should contain only one Goldstone superfield (with the Goldstone fermion among their components). The rest of the fields must be matter ones, i.e. they should not transform with respect to broken supersymmetry. Let us demonstrate how such a splitting works in the present system.

3.1 The case with $d_{111} = d_{222} = 1$

With such a choice of the symmetric tensor d_{abc} , the basic constraints (2.11), (2.13) have a splitting form

$$\begin{cases} \psi_1 D\psi_1 - \nu_1 (m_1 - D\nu_1) = 0, & \psi_1 \nu_1 = 0, \\ \psi_2 D\psi_2 - \nu_2 (m_2 - D\nu_2) = 0, & \psi_2 \nu_2 = 0. \end{cases} \quad (3.1)$$

By rescaling of the variables ψ_a and ν_a in (3.1), one may always choose

$$m_1 = m_2 = 1. \quad (3.2)$$

The transformation properties of ψ_a and ν_a (2.10) with such a choice of parameters m_a are

$$\begin{cases} \delta\psi_1 = \epsilon (1 - D\nu_1), & \delta\nu_1 = \epsilon D\psi_1, \\ \delta\psi_2 = \epsilon (1 - D\nu_2), & \delta\nu_2 = \epsilon D\psi_2. \end{cases} \quad (3.3)$$

Clearly, the solution of the constraints (3.1) is quite similar to the one particle case (2.7) and now it reads

$$\nu_1 = \frac{2\psi_1 D\psi_1}{1 + \sqrt{1 - 4(D\psi_1)^2}}, \quad \nu_2 = \frac{2\psi_2 D\psi_2}{1 + \sqrt{1 - 4(D\psi_2)^2}}. \quad (3.4)$$

Let us now introduce *two* Goldstone spinor superfields ξ_a with the following transformation properties

$$\delta\xi_1 = \epsilon + \epsilon \xi_1 \partial_t \xi_1, \quad \delta\xi_2 = \epsilon + \epsilon \xi_2 \partial_t \xi_2. \quad (3.5)$$

It is easy to check that the tilded fields $\tilde{\psi}_a$ and $\tilde{\nu}_a$ defined as

$$\tilde{\psi}_a = \psi_a - \xi_a (1 - D\nu_a), \quad \tilde{\nu}_a = \nu_a - \xi_a D\psi_a, \quad (3.6)$$

transform under (3.3), (3.5) as (see, e.g. [11])

$$\delta\tilde{\psi}_a = \epsilon \xi_a \partial_t \tilde{\psi}_a, \quad \delta\tilde{\nu}_a = \epsilon \xi_a \partial_t \tilde{\nu}_a. \quad (3.7)$$

Thus, it is a covariant operation to put these superfields equal to zero

$$\begin{cases} \tilde{\psi}_a = 0, & \psi_a - \xi_a (1 - D\nu_a) = 0, \\ \tilde{\nu}_a = 0 & \Rightarrow \quad \nu_a - \xi_a D\psi_a = 0. \end{cases} \quad (3.8)$$

Moreover, the constraints (3.1) immediately follow from (3.8). One should note that the constraints $\tilde{\psi}_a = \tilde{\nu}_a = 0$ contain some additional information: they can be used to express the superfields of the linear realization ψ_a and ν_a in terms of the Goldstone superfields ξ_a

$$\nu_a = \frac{\xi_a D\xi_a}{1 + D\xi_a D\xi_a}, \quad \psi_a = \frac{\xi_a}{1 + D\xi_a D\xi_a} \quad (\text{no summation over } a). \quad (3.9)$$

Until now we have two Goldstone superfields ξ_a , while we are expecting to have only one essential Goldstone fermionic superfield and one matter superfield. This may be achieved by passing to the new superfields η and λ

$$\eta = \frac{1}{2} (\xi_1 + \xi_2) + \frac{1}{4} \xi_1 \xi_2 (\dot{\xi}_1 - \dot{\xi}_2), \quad \lambda = \frac{1}{2} (\xi_1 - \xi_2) + \frac{1}{4} \xi_1 \xi_2 (\dot{\xi}_1 + \dot{\xi}_2), \quad (3.10)$$

which, in virtue of (3.5), transform as expected³

$$\delta\eta = \epsilon + \epsilon\eta\partial_t\eta, \quad \delta\lambda = \epsilon\eta\partial_t\lambda. \quad (3.11)$$

From (3.11) one concludes that η is the fermionic Goldstone superfield accompanying the present spontaneous breaking of $N = 2$ supersymmetry, while λ is a matter fermionic superfield. The superfields ξ_a as well as ψ_a, ν_a can be expressed in the terms of η and λ :

$$\xi_1 = \eta + \lambda + \eta\lambda(\dot{\eta} + \dot{\lambda}), \quad \xi_2 = \eta - \lambda - \eta\lambda(\dot{\eta} - \dot{\lambda}), \quad (3.12)$$

and

$$\begin{aligned} \psi_1 &= \frac{\eta + \lambda}{1 + (D\eta + D\lambda)^2} + \eta\lambda(\dot{\eta} + \dot{\lambda}) \frac{1 - (D\eta + D\lambda)^2}{[1 + (D\eta + D\lambda)^2]^2}, & \psi_2 &= \psi_1|_{\lambda \rightarrow -\lambda}, \\ \nu_1 &= (D\eta + D\lambda) \left[\frac{\eta + \lambda}{1 + (D\eta + D\lambda)^2} + \frac{2\eta\lambda(\dot{\eta} + \dot{\lambda})}{[1 + (D\eta + D\lambda)^2]^2} \right], & \nu_2 &= \nu_1|_{\lambda \rightarrow -\lambda}. \end{aligned} \quad (3.13)$$

Thus, the system of constraints (3.1) represents a non-standard description of the essential Goldstone superfield η and a matter fermionic superfield λ .

Clearly, the FPS-like action (1.7) for this case reads

$$S = \int dt d\theta \left\{ e_1 \nu_1 + e_2 \nu_2 + C_{12}(\psi_1 D\psi_2 - \nu_1(1 - D\nu_2) + \psi_2 D\psi_1 - \nu_2(1 - D\nu_1)) \right\} \quad (3.14)$$

and it is invariant with respect to both supersymmetries.

3.2 The case with $d_{111} = 1, d_{122} = -1$

With this choice of constants the constraints (2.11) read

$$\begin{aligned} \psi_1 D\psi_1 - \psi_2 D\psi_2 - \nu_1(m_1 - D\nu_1) + \nu_2(m_2 - D\nu_2) &= 0, \\ \psi_1 D\psi_2 + \psi_2 D\psi_1 - \nu_1(m_2 - D\nu_2) - \nu_2(m_1 - D\nu_1) &= 0, \end{aligned} \quad (3.15)$$

while those in (2.13) take form

$$\psi_1 \nu_1 - \psi_2 \nu_2 = 0, \quad \psi_1 \nu_2 + \psi_2 \nu_1 = 0. \quad (3.16)$$

One can introduce the complex superfields ψ, ν and complex parameter m as

$$\begin{aligned} \psi &= \psi_1 + i\psi_2, & \nu &= \nu_1 + i\nu_2, & m &= m_1 + im_2, \\ \bar{\psi} &= \psi_1 - i\psi_2, & \bar{\nu} &= \nu_1 - i\nu_2, & \bar{m} &= m_1 - im_2, \end{aligned} \quad (3.17)$$

and then rewrite eqs. (3.15) and (3.16) in the splitting form [10]

$$\begin{aligned} \psi D\psi - \nu(m - D\nu) &= 0, & \psi \nu &= 0, \\ \bar{\psi} D\bar{\psi} - \bar{\nu}(\bar{m} - D\bar{\nu}) &= 0, & \bar{\psi} \bar{\nu} &= 0. \end{aligned} \quad (3.18)$$

³This is the form-variation of the fields under implicit $N = 1$ supersymmetry: $\delta\mathcal{A} = \mathcal{A}'(t, \theta) - \mathcal{A}(t, \theta)$.

The equations (3.18) have the evident solution

$$\nu = \frac{2\psi D\psi}{m + \sqrt{m^2 - 4(D\psi)^2}}, \quad \bar{\nu} = \frac{2\bar{\psi} D\bar{\psi}}{\bar{m} + \sqrt{\bar{m}^2 - 4(D\bar{\psi})^2}}. \quad (3.19)$$

Finally, the FPS-like action (1.7) reads

$$S = \int dt d\theta \left\{ e \nu + e^* \bar{\nu} + C_{12} (\psi D\bar{\psi} - \nu(\bar{m} - D\bar{\nu}) + \bar{\psi} D\psi - \bar{\nu}(m - D\nu)) \right\}, \quad (3.20)$$

where $e = \frac{1}{2}(e_1 - ie_2)$, $e^* = \frac{1}{2}(e_1 + ie_2)$. This action is also invariant with respect to both supersymmetries.

One should note, that one may always choose the constants m_1, m_2 as

$$m_1 = 1, \quad m_2 = 0, \quad \Rightarrow \quad m = \bar{m} = 1. \quad (3.21)$$

The choice $m_2 = 0$ is achieved by passing to the fields ρ, μ

$$\psi_2 \rightarrow \rho = m_2 \psi_1 - m_1 \psi_2, \quad \mu = m_2 \nu_1 - m_1 \nu_2,$$

and, as it follows from (2.10)

$$\delta\rho = -\varepsilon D\mu, \quad \delta\mu = \varepsilon D\rho. \quad (3.22)$$

The choice $m_1 = 1$ is guaranteed by the rescaling in (2.10)

$$\psi_1 \rightarrow \tilde{\psi} = m_1 \psi_1, \quad \nu_1 \rightarrow \tilde{\nu} = m_1 \nu_1.$$

Now we have a full analogy with the previously considered splitting case. Indeed, the complex superfields ψ and ν transform with respect to broken supersymmetry as

$$\begin{cases} \delta\psi = \epsilon(1 - D\nu), & \delta\nu = \epsilon D\psi, \\ \delta\bar{\psi} = \epsilon(1 - D\bar{\nu}), & \delta\bar{\nu} = \epsilon D\bar{\psi}. \end{cases} \quad (3.23)$$

and obey the constraints

$$\begin{cases} \psi D\psi - \nu(1 - D\nu) = 0, & \psi \nu = 0 \\ \bar{\psi} D\bar{\psi} - \bar{\nu}(1 - D\bar{\nu}) = 0, & \bar{\psi} \bar{\nu} = 0 \end{cases} \quad (3.24)$$

If we now introduce *two* Goldstone spinor superfields $\xi, \bar{\xi}$ with the following transformation properties

$$\delta\xi = \epsilon + \epsilon \xi \partial_t \xi, \quad \delta\bar{\xi} = \epsilon + \epsilon \bar{\xi} \partial_t \bar{\xi}, \quad (3.25)$$

than the tilded fields $\tilde{\psi}, \tilde{\bar{\psi}}$ and $\tilde{\nu}, \tilde{\bar{\nu}}$, which are defined as

$$\tilde{\psi} = \psi - \xi(1 - D\nu), \quad \tilde{\nu} = \nu - \xi D\psi, \quad \tilde{\bar{\psi}} = \bar{\psi} - \bar{\xi}(1 - D\bar{\nu}), \quad \tilde{\bar{\nu}} = \bar{\nu} - \bar{\xi} D\bar{\psi}, \quad (3.26)$$

transform under (3.23), (3.25) as

$$\delta\tilde{\psi} = \epsilon \xi \partial_t \tilde{\psi}, \quad \delta\tilde{\nu} = \epsilon \xi \partial_t \tilde{\nu}, \quad \delta\tilde{\bar{\psi}} = \epsilon \bar{\xi} \partial_t \tilde{\bar{\psi}}, \quad \delta\tilde{\bar{\nu}} = \epsilon \bar{\xi} \partial_t \tilde{\bar{\nu}}. \quad (3.27)$$

Thus, again there is a covariant operation to put these superfields equal to zero

$$\begin{cases} \tilde{\psi} = 0, & \tilde{\nu} = 0, & \psi - \xi(1 - D\nu) = 0, & \nu - \xi D\psi = 0, \\ \bar{\tilde{\psi}} = 0, & \bar{\tilde{\nu}} = 0 & \Rightarrow & \bar{\psi} - \bar{\xi}(1 - D\bar{\nu}) = 0, & \bar{\nu} - \bar{\xi} D\bar{\psi} = 0. \end{cases} \quad (3.28)$$

Clearly, the constraints (3.24) immediately follow from (3.28). The constraints (3.28) can be easily solved

$$\nu = \frac{\xi D\xi}{1 + D\xi D\xi}, \quad \psi = \frac{\xi}{1 + D\xi D\xi}, \quad \bar{\nu} = \frac{\bar{\xi} D\bar{\xi}}{1 + D\bar{\xi} D\bar{\xi}}, \quad \bar{\psi} = \frac{\bar{\xi}}{1 + D\bar{\xi} D\bar{\xi}}. \quad (3.29)$$

Repeating all calculations from previous subsection we will get that the superfields η and λ defined as

$$\eta = \frac{1}{2}(\xi + \bar{\xi}) + \frac{1}{4}\xi\bar{\xi}\left(\dot{\xi} - \dot{\bar{\xi}}\right), \quad \lambda = \frac{i}{2}(\xi - \bar{\xi}) + \frac{i}{4}\xi\bar{\xi}\left(\dot{\xi} + \dot{\bar{\xi}}\right), \quad (3.30)$$

have the same transformation properties as in (3.11)

$$\delta\eta = \epsilon + \epsilon\eta\partial_t\eta, \quad \delta\lambda = \epsilon\eta\partial_t\lambda.$$

It is important that the transformations (3.30) are invertible and the superfields $\xi, \bar{\xi}$ may be expressed in terms of η, λ as

$$\xi = \eta - i\lambda - i\eta\lambda\left(\dot{\eta} - i\dot{\lambda}\right), \quad \bar{\xi} = \eta + i\lambda + i\eta\lambda\left(\dot{\eta} + i\dot{\lambda}\right). \quad (3.31)$$

Finally, the genuine superfields ψ, ν can be expressed in terms of the Goldstone superfield η and matter superfield λ :

$$\begin{aligned} \psi &= \frac{\eta - i\lambda}{1 + (D\eta - iD\lambda)^2} - i\eta\lambda\left(\dot{\eta} - i\dot{\lambda}\right) \frac{1 - (D\eta - iD\lambda)^2}{\left[1 + (D\eta - iD\lambda)^2\right]^2}, & \bar{\psi} &= (\psi)^\dagger, \\ \nu &= (D\eta - iD\lambda) \left[\frac{\eta - i\lambda}{1 + (D\eta - iD\lambda)^2} - i \frac{2\eta\lambda\left(\dot{\eta} - i\dot{\lambda}\right)}{\left[1 + (D\eta - iD\lambda)^2\right]^2} \right], & \bar{\nu} &= (\nu)^\dagger. \end{aligned} \quad (3.32)$$

3.3 Relations between two cases

Having at hands the *invertible* relations between ξ_1, ξ_2 and η, λ (3.12) and between η, λ and $\xi, \bar{\xi}$ (3.30), one may easily find the relations between $\xi, \bar{\xi}$ and ξ_1, ξ_2

$$\begin{aligned} \xi &= \frac{1}{2}(1 - i)\xi_1 + \frac{1}{2}(1 + i)\xi_2 + \frac{1}{2}\xi_1\xi_2\left(\xi_1 - \xi_2\right), & \bar{\xi} &= \frac{1}{2}(1 + i)\xi_1 + \frac{1}{2}(1 - i)\xi_2 + \frac{1}{2}\xi_1\xi_2\left(\xi_1 - \xi_2\right), \\ \xi_1 &= \frac{1}{2}(1 + i)\xi + \frac{1}{2}(1 - i)\bar{\xi} + \frac{1}{2}\xi\bar{\xi}\left(\dot{\xi} - \dot{\bar{\xi}}\right), & \xi_2 &= \frac{1}{2}(1 - i)\xi + \frac{1}{2}(1 + i)\bar{\xi} + \frac{1}{2}\xi\bar{\xi}\left(\dot{\xi} - \dot{\bar{\xi}}\right). \end{aligned} \quad (3.33)$$

Thus, at least in one dimension, the two cases are completely equivalent, because they are related by *invertible* fields redefinition (3.33). The only difference between these cases lies in the different definition of the invariant actions (3.20). Moreover, these two actions, being clearly related in view of (3.33), are not the most general ones. In the next section we will construct the most general action for the Goldstone fermion η and the matter fermionic superfield λ .

4 Nonlinear realization approach

An alternative description of the discussed system with partially broken $N = 2, d = 1$ supersymmetry may be provided by the nonlinear realizations approach. It turns out that in the present case this approach is more suitable for the construction of the most general superfield action. Let us demonstrate how such an action can be derived.

4.1 Key ingredients

We start with the $N = 2, d = 1$ Poincaré superalgebra with one central charge generator Z

$$\{Q, Q\} = 2P, \quad \{S, S\} = 2P, \quad \{Q, S\} = 2Z. \quad (4.1)$$

Introducing a coset element g as

$$g = e^{tP} e^{\theta Q} e^{qZ} e^{\eta S}, \quad (4.2)$$

and calculating the expression $g^{-1}dg = \omega_P P + \omega_Q Q + \omega_Z Z + \omega_S S$, one finds the explicit expressions for the Cartan forms

$$\omega_P = dt - d\theta\theta - d\eta\eta, \quad \omega_Q = d\theta, \quad \omega_Z = dq - 2d\theta\eta, \quad \omega_S = d\eta. \quad (4.3)$$

The covariant derivatives can be found in a standard way, and in the present case they read

$$\nabla_\theta = D + \eta D\eta \nabla_t, \quad \nabla_t = E^{-1} \partial_t, \quad (4.4)$$

where

$$E = 1 + \eta \partial_t \eta, \quad E^{-1} = 1 - \eta \nabla_t \eta. \quad (4.5)$$

These derivatives satisfy the following (anti)commutation relations

$$\{\nabla_\theta, \nabla_\theta\} = 2(1 + \nabla_\theta \eta \nabla_\theta \eta) \nabla_t, \quad [\nabla_t, \nabla_\theta] = 2\nabla_t \eta \nabla_\theta \eta \nabla_t. \quad (4.6)$$

Acting on (4.2) from the left by different elements of the $N = 2, d = 1$ Poincaré supergroup g_0 one can find the transformation properties of the coordinates (t, θ) and the Goldstone superfield $\eta(t, \theta)$. In particular, under both unbroken and broken supersymmetries they transform as follows:

- Unbroken supersymmetry ($g_0 = e^{\epsilon Q}$):

$$\delta_Q t = -\epsilon\theta, \quad \delta_Q \theta = \epsilon, \quad (4.7)$$

- Broken supersymmetry ($g_0 = e^{\varepsilon S}$):

$$\delta_S t = -\varepsilon\eta, \quad \delta_S q = -2\varepsilon\theta, \quad \delta_S \eta = \varepsilon. \quad (4.8)$$

The final step is to express the superfield η in terms of the bosonic superfield q by imposing the constraint [12]

$$\omega_Z|_{d\theta} = 0 \quad \Rightarrow \quad \eta = \frac{1}{2} \nabla_\theta q. \quad (4.9)$$

It should be noted that in order to describe the matter superfield λ within the nonlinear realization approach, one should postulate its transformation properties under S supersymmetry to be⁴

$$\delta_S \lambda = 0. \quad (4.10)$$

One may also define the bosonic superfield ϕ with the transformation property

$$\delta_S \phi = 0, \quad (4.11)$$

which is related with λ in the same manner as in (4.9)

$$\lambda = \frac{1}{2} \nabla_\theta \phi. \quad (4.12)$$

4.2 General action

The most general Ansatz for the $N = 1$ superfield action describing the Goldstone fermionic superfield η and the matter fermionic superfield λ and having no dimensional constants reads

$$S = \int dt d\theta (1 + \eta \dot{\eta}) [\eta F_1 + \lambda F_2 + \eta \dot{\eta} \lambda F_3 + \eta \lambda \nabla_t \lambda F_4], \quad (4.13)$$

where F_1, F_2, F_3, F_4 are arbitrary functions depending on $\nabla_\theta \eta$ and $\nabla_\theta \lambda$, only. The reason for such a form of Ansatz is quite understandable:

- the functions F_1, F_2, F_3, F_4 have to be defined by imposing the invariance of the action (4.13) with respect to broken S -supersymmetry
- with respect to S supersymmetry, $\delta_{St} = -\varepsilon \eta$, $\delta_S \eta = \varepsilon$, $\delta_S \lambda = 0$, the “improved” measure $dt d\theta (1 + \eta \dot{\eta})$ is invariant
- the functions F_1, F_2, F_3, F_4 as well as $\nabla_t \lambda$ are also invariant with respect to S -supersymmetry.

Therefore, the variation of the action (4.13) with respect to broken supersymmetry has a very simple form

$$\delta_S S = \int dt d\theta \varepsilon \left[(1 + \eta \dot{\eta}) F_1 + \dot{\eta} \lambda F_3 + \lambda \dot{\lambda} F_4 \right]. \quad (4.14)$$

Thus, the function F_2 may be chosen to be an arbitrary function of its arguments. To fix the functions F_1, F_3 and F_4 from the condition $\delta_S S = 0$ one has, firstly, to integrate⁵ over θ in (4.14) and then to replace the arguments of these functions $\nabla_\theta \eta, \nabla_\theta \lambda$ by

$$\nabla_\theta \eta = (1 + \eta \dot{\eta}) D\eta \equiv (1 + \eta \dot{\eta}) x, \quad \nabla_\theta \lambda = D\lambda + \eta \dot{\lambda} D\eta \equiv y + \eta \dot{\lambda} x. \quad (4.15)$$

⁴Due to the transformation of the time t in (4.8) the form-variation of λ will be $\delta \lambda = \varepsilon \eta \partial_t \lambda$.

⁵The problem is that the integrand in (4.14) can be represented as a D derivative acting on some function, thus yielding a full time derivative after hitting the resulting expression with the spinor derivative coming from the measure. Such a condition is slightly more complicated for analysis than the one which follows after integration over θ .

Therefore

$$F[\nabla_\theta \eta, \nabla_\theta \lambda] = F[x, y] + \eta \dot{\eta} x F[x, y]_x + \eta \dot{\lambda} x F[x, y]_y, \quad (4.16)$$

where

$$x \equiv D\eta, \quad y \equiv D\lambda. \quad (4.17)$$

Performing the integration in (4.14), expanding the functions F_1, F_3, F_4 as in (4.16) and collecting the terms which are linear and cubic in the fermions we will get three terms

$$\dot{\eta} [x F_1 + (1 + x^2) (F_1)_x - y F_3] - \eta \frac{d}{dt} [x F_1], \quad (4.18)$$

$$\dot{\lambda} [(1 + x^2) (F_1)_y + y F_4] - \lambda (\dot{y} F_4 - \dot{x} F_3), \quad (4.19)$$

$$\left((F_3)_y + (F_4)_x \right) \left(\lambda \dot{\lambda} \dot{\eta} + x \left(x \lambda \dot{\lambda} \dot{\eta} - \eta \lambda \dot{\lambda} \dot{x} + \eta \dot{\eta} \lambda \dot{y} - \eta \dot{\eta} \lambda \dot{y} \right) \right), \quad (4.20)$$

which have to be equal to zero after integration over t , independently. The simplest way to get the corresponding equations is to consider the equations of motion for η and λ which follow from (4.18)–(4.20). Thus, we have three equations

$$[(1 + x^2) F_1]_x - y F_3 = a, \quad (a)$$

$$y [(1 + x^2) F_1]_{yy} + [y^2 F_4]_y = 0, \quad (b) \quad (4.21)$$

$$(F_3)_y + (F_4)_x = 0, \quad (c)$$

where a in the first equation is an arbitrary constant.

The equation (4.21b) may be integrated once, giving

$$y (\tilde{F}_1)_y - \tilde{F}_1 + y^2 F_4 = G[x], \quad (4.22)$$

where

$$\tilde{F}_1 \equiv (1 + x^2) F_1, \quad (4.23)$$

and $G[x]$ is an arbitrary function depending on x only. Note, that the equation (4.21a) now reads

$$(\tilde{F}_1)_x - y F_3 = a. \quad (4.24)$$

Differentiating the equation (4.22) over x and the equation (4.24) over y , and using the equation (4.21c) one may get

$$G'[x] = -a \quad \rightarrow \quad G[x] = -ax - b, \quad (4.25)$$

where b is a new constant.

Finally, representing the function \tilde{F}_1 as

$$\tilde{F}_1 = ax + b + y \hat{F}_1 \quad (4.26)$$

we will finish with the equations

$$(\hat{F}_1)_y + F_4 = 0, \quad (\hat{F}_1)_x - F_3 = 0, \quad (4.27)$$

which define the functions F_3, F_4 in terms of an arbitrary function \hat{F}_1 .

Thus, the action (4.13), with the functions

$$F_1 = \frac{ax+b}{1+x^2} + \frac{y}{1+x^2} \hat{F}_1, \quad F_3 = \left(\hat{F}_1 \right)_x, \quad F_4 = - \left(\hat{F}_1 \right)_y \quad (4.28)$$

and arbitrary functions \hat{F}_1 and F_2 , is invariant under the broken S supersymmetry. The last step is to note that the action (4.13) with $F_1 = \frac{b}{1+x^2}$ is trivial, because

$$S_{triv} = \int dt d\theta (1 + \eta \dot{\eta}) \frac{\eta}{1 + (\nabla_\theta \eta)^2} = \int dt \partial_t q = 0 \quad (4.29)$$

in view of (4.9). Thus, the constant b is unessential and the final result for the functions in the action (4.13) reads

$$F_1 = a \frac{\nabla_\theta \eta}{1 + (\nabla_\theta \eta)^2} + \frac{\nabla_\theta \lambda}{1 + (\nabla_\theta \eta)^2} \hat{F}_1, \quad F_3 = \left(\hat{F}_1 \right)_{\nabla_\theta \eta}, \quad F_4 = - \left(\hat{F}_1 \right)_{\nabla_\theta \lambda}, \quad (4.30)$$

with \hat{F}_1 and F_2 being arbitrary functions of $\nabla_\theta \eta$ and $\nabla_\theta \lambda$

$$\hat{F}_1 = \hat{F}_1[\nabla_\theta \eta, \nabla_\theta \lambda], \quad F_2 = F_2[\nabla_\theta \eta, \nabla_\theta \lambda]. \quad (4.31)$$

5 Interesting cases

Having at hands the most general expression for the action invariant with respect to both Q and S supersymmetries (4.13), (4.30), it is interesting to visualize the functions F_1, F_2 which being substituted into (4.13) will reproduce the FPS actions (3.14), (3.20). Comparing (3.13) and (3.32) we may conclude, that these two FPS cases are related through the substitutions $\lambda \rightarrow -i\lambda, y \rightarrow -iy$. Therefore, it is enough to consider the first case, with $d_{111} = d_{222} = 1$, only. To simplify everything, it is useful to represent the integrand in (4.13) as follows

$$L = \eta F_1 + \lambda F_2 + \lambda \eta \dot{\eta} \left(x F_2 + \hat{F}_1 \right)_x - \eta \lambda \dot{\lambda} \left(x F_2 + \hat{F}_1 \right)_y, \quad (5.1)$$

with

$$F_1 \equiv a \frac{x}{1+x^2} + \frac{y}{1+x^2} \hat{F}_1, \quad (5.2)$$

where the functions \hat{F}_1 and F_2 depend on x, y variables (4.17). Comparing the integrands in the actions (3.14) and (5.1), we will get

$$L_{\text{FPS}} = \nu_1 \Rightarrow F_1 = F_2 = \frac{x+y}{1+(x+y)^2}, \quad a = 1, \quad (5.3)$$

$$L_{\text{FPS}} = \psi_1 D\psi_2 + \nu_1 D\nu_2 \Rightarrow F_1 = F_2 = \frac{(x-y)(1+x^2-y^2)}{(1+(x-y)^2)(1+(x+y)^2)}, \quad a = 1. \quad (5.4)$$

Unfortunately, the explicit form of the functions F_1, F_2 which corresponds to FPS-like actions (5.3), (5.4) is not informative enough to understand why these actions were selected. The action (4.13), with the restrictions (4.30), contains two arbitrary functions F_1 and F_2 . So, as we expected, the invariance with respect to additional, spontaneously broken $N = 1$

supersymmetry does not fix the action in the many particles case, in contrast with the one particle case, which corresponds to the conditions $\hat{F}_1 = F_2 = 0$ in the superfield Lagrangian (5.1). The bosonic core of the action (4.13) with the Lagrangian (5.1) has the form

$$S_{\text{bos}} = \int dt \frac{1}{1 + \sqrt{1 - \dot{q}^2}} \left(\dot{q} \mathcal{F}_1 + \dot{\phi} \mathcal{F}_2 \right), \quad (5.5)$$

where \mathcal{F}_1 and \mathcal{F}_2 are arbitrary functions depending on \dot{q} and $\dot{\phi}$.

The explicit form of the Lagrangian in (5.1) suggests one special case with

$$\hat{F}_1 = -x F_2 \quad (5.6)$$

for which the terms cubic in the fermions disappear. With such a choice, the bosonic core simplifies to be

$$\hat{S}_{\text{bos}} = \int dt \left(\frac{a}{2} \left(1 - \sqrt{1 - \dot{q}^2} \right) + \frac{1}{2} \dot{\phi} \mathcal{F}_2 \right). \quad (5.7)$$

Now, choosing the function \mathcal{F}_2 to be

$$\mathcal{F}_2 = \frac{b \dot{\phi}}{1 + \sqrt{1 - \dot{\phi}^2}} \quad (5.8)$$

we will get⁶

$$\tilde{S}_{\text{bos}} = \int dt \left(\frac{a}{2} \left(1 - \sqrt{1 - \dot{q}^2} \right) + \frac{b}{2} \left(1 - \sqrt{1 - \dot{\phi}^2} \right) \right). \quad (5.9)$$

This case is indeed special, but again we have no any arguments to clarify this choice.

6 Conclusion

In this paper we analysed in details the application of the FPS approach in one dimension. The basic steps were the same as in the paper [1]. The resulting system is described by the $N = 2, d = 1$ supersymmetric action for two particles in which one of $N = 1$ supersymmetries is spontaneously broken. The final actions possess the same features as the FPS ones. Using the nonlinear realization approach we reconsider the system in the basis where only one superfield has the Goldstone nature while the second superfield can be treated as the matter one, being invariant under transformations of the spontaneously broken $N = 1$ supersymmetry. Having at hands the transformations relating the two selected FPS-like cases with our more generic one, we established the field redefinitions which relate these two cases. Thus, the only difference between two FPS cases lies in the different choice of the actions, while the supermultiplets specified by the FPS-like constraints are really the same. In our basis, the two FPS supermultiplets are related by the redefinition of the matter superfield $\lambda \rightarrow -i\lambda$. Therefore, it becomes clear why the action with the right signs of the kinetic terms for the bosonic components in one case is mapped, being rewritten in terms of the second supermultiplet, into the action with the wrong sign of the kinetic term for one of the bosonic field. Going further with the nonlinear realization approach, we

⁶One should remind that with our definitions (4.9) and (4.12), the bosonic limits of the variables x, y are related with the time derivatives of q and ϕ as $x = \frac{\dot{q}}{1 + \sqrt{1 - \dot{q}^2}}, y = \frac{\dot{\phi}}{1 + \sqrt{1 - \dot{\phi}^2}}$.

constructed the most general action for the system of two $N = 1$ superfields possessing one additional hidden spontaneously broken $N = 1$ supersymmetry. The constructed action contains two arbitrary functions and is reduced to the FPS actions upon specification of these functions. Unfortunately, the exact form of these functions corresponding to FPS actions is not very informative and it gives no reason why the FPS cases were selected.

Of course, our consideration was strictly one-dimensional and therefore we cannot argue that all features we discussed will appear in the generalized supersymmetric Born-Infeld theory constructed in [1, 3]. Nevertheless, we believe that our results and the generality of the nonlinear realization approach are quite reasonable tools to reconsider the questions:

- whether the supermultiplets with different constants d_{abc} are really different?
- which additional properties select the FPS-like actions?

in four dimensions.

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